A New Explanation of Pitch Perception Phenomena described in Literature.

With the new Hearing Paradigm where the incoming sound in the cochlea is Differentiated and Squared.

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Summary.

1. Introduction.

In the paper of Heerens & De Ru (2010)[1] it is explained that the human hearing sense is differentiating and squaring the incoming sound pressure stimulus in the cochlea. This because the sound pressure in front of the eardrum is transferred to the velocity in the perilymph fluid content of the duct, that is apically folded over 180° at the helicotrema and that exist of the scala vestibuli (at its base enclosed by the oval window) and scala tympani (at its base enclosed by the round window).

Perilymph is a low viscous fluid, practically incompressible, in which the sound velocity is 1500 m/s, hence for all acoustic frequencies the length of the scala tympani is much smaller than the wavelength of concerned contributions, and the perilymph column is moving as a whole. While even for the highest possible perilymph velocities the movement is laminar, it can be considered as a quasi-static flow. And then it is allowed to apply Bernoulli's law for fluid flow, so the pressure induced on the wall of the duct, including the basilar membrane, is proportional to the perilymph velocity squared.

This means that the evoked signal in the organ of Corti is representative for the sound energy signal and not for the sound pressure signal as is commonly thought.

The conclusions that can be drawn out of the experiments of Wever & Lawrence reported in their paper of (1950)[2] are in complete agreement with the statement that the relation between incoming sound signal and evoked cochlear potentials in the cochlea is similar to a transfer of the sound energy frequency spectrum via the auditory nerve to the brain.

Next to that the paper of Heerens & De Ru[1] also explains that the extreme small changes in pressure from normal sound signals are not capable of a measurable deformation of the petrous bone in which the cochlea is situated. Instead of that it is highly likely that 'bone' conduction actually is a perilymph push-pull movement out of the cerebrospinal cavity via the connecting duct – cochlear aqueduct – evoked by small deformations over larger areas the of the shell shaped bones of the skull.

But also there exist the serious possibility that perilymph movement in the one ear causes perilymph push-pull via the two cochlear aqueducts and the cerebrospinal cavity and finally a perilymph movement in the other ear.

Because the cochlear aqueduct is connected with the scala tympani close to the round window, the perilymph movement out of the cerebrospinal cavity will split in the two opposite directions, one towards the round window, so not along the basilar membrane, and the other in apical direction along the basilar membrane. Since such a movement implies that a perilymph push-pull velocity is evoked along the basilar membrane, it will also contribute to the total stimulus for the cochlea concerned.

The utmost important consequence of this is that the non-linear behavior of the cochlea is exactly quadratic. Both for monaural and binaural acoustic stimuli. And this opens tremendous new possibilities for explaining at least the mysterious pitch phenomena that are described by Alain de Cheveigné[3] in his draft chapter (2008) for the book: Plack, C. 'Auditory Perception (Oxford University Press Handbook of Auditory Science), scheduled for publication in 2008 or 2009 but not yet published. I will use this chapter as guide for the explanation of a number of pitch phenomena. Even mysterious ones.

In that chapter De Cheveigné explains what pitch is and why it is puzzling so many researchers. I cite him here:

--- To summarize, pitch is a very important aspect of sound perception. We can discriminate exquisitely small differences in pitch, while ignoring salient differences along other perceptual dimensions. Pitch has more to it than the simple, one-dimensional construct assumed by psychophysics, and yet we have few models to account for these complexities. That so much is yet unknown about pitch is sobering for those of us who have been working on it for years, and exhilarating for whoever sets out to search for more: there's lots more to discover! ---

As will be shown in the next paragraphs such an exhilarating breakthrough in research, based on that new paradigm, is presented at first for harmonic tone complexes.

2. Harmonic Tone Complexes Generating Pitches.

In the introductory paragraph of his chapter De Cheveigné describes the following:

--- For the psycho-acoustician, pitch is the perceptual correlate of fundamental frequency ($\boldsymbol{F0}$), that is, the rate at which a periodic waveform repeats itself. A periodic sound produces a pitch that depends on the period $\boldsymbol{T} = 1/\boldsymbol{F0}$: the shorter the period, the higher the pitch. The quantitative relation between period of vibration and notes of the musical scale was established early in the 17th century by Mersenne and Galileo (see de Cheveigné 2005 for a review). ---

The cardinal relationship mentioned here, will be explained later. The introduction continues with the following:

--- It is customary in psychoacoustics to distinguish *pure tones*, with a sinusoidal waveform and a single-component spectrum, from *complex tones* with a waveform that is arbitrarily shaped but nevertheless periodic, and a spectrum with multiple components that are harmonically related (i.e. all multiples of the same $\boldsymbol{F0}$). Much past research on pitch has focused on pure tones under the belief that the percept that they evoke is somehow "elementary". Here we treat the pure tone as one among the many stimuli that may evoke a pitch. ---

And then there follows a number of illustrated examples of pitch combined with an illustrative figure, that I have split into 6 separate illustrations for the different forms of a pitch related to the fundamental based on periodicity.

For each of them I will compare the given description with the explanation based on the new hearing paradigm. Here we have to consider that all these pitches are commonly supposed to be developed in the brain. I will show you with the new paradigm that the pitch is solely evoked inside the cochlea on the basilar membrane.

2.a. A single pure tone.

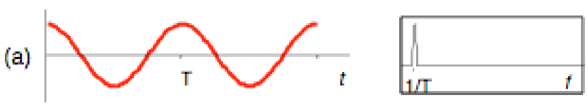

Fig.1. Waveform and spectrum of a single pure tone [example a from De Cheveigné[3]]

The first example (a) is at first sight trivial:

The pure tone, a sinusoidal pressure stimulus with frequency f, or a period $T = 1/f$ and pressure amplitude p_0.

This according to the equation:

$$p = p_0 \sin(2\pi f t) = p_0 \sin(2\pi\, t/T) \qquad (1)$$

Due to the attenuation with a factor A in the transfer from eardrum to oval window deflection and differentiation $d\,/\,dt$ to transfer this in the perilymph velocity, that velocity will be expressed by:

$$v = v_0 \cos(2\pi f t) = 2\pi f A\, p_0 \cos(2\pi f t) \qquad (2)$$

Using Bernoulli's equation the pressure on the basilar membrane will be given by:

$$\Delta p = -\tfrac{1}{2}\rho\, v^2 = -2\rho\,(\pi f A\, p_0)^2 \cos^2(2\pi f t). \qquad (3)$$

This can be written as:

$$\Delta p = \Delta p_0 \left[\tfrac{1}{2} + \tfrac{1}{2}\cos(4\pi f t)\right], \qquad (4)$$

with:

$$\Delta p_0 = -2\rho\,(\pi f A\, p_0)^2 \qquad (5)$$

which means that on the basilar membrane will be evoked a doubled frequency next to a constant contribution.

But there is more to conclude.

What also can be observed is that if we take the sound pressure amplitude p_0 constant, and we vary the frequency, the stimulus on the basilar membrane will be proportional to the frequency squared. Or a frequency dependent sensitivity increase with 6 dB/octave.

This is also a direct indication why the human hearing sense prefers music and sounds that are built-up with the so-called $1/f$ frequency dependency for the amplitude of sound pressure contributions. Such a $1/f$ dependency accomplishes equality in sound energy contributions of individual pure tones and makes that 'pink' noise sound pressure contributions evoke 'white' noise sound energy stimuli on the basilar membrane.

And since the Fletcher-Munson sensitivity curve is measured with constant sound pressure stimuli, this is also the reason that a -6 dB/octave correction creates a flattening of the sensitivity over a wide frequency interval between 100 Hz and 4 kHz

In order to normalize the other examples for equal sound energy contribution we make the choice that the sound pressure amplitude $p_0(f)$ for the frequency f contribution in relation to the sound pressure amplitude $p_0(f_0)$ for frequency f_0 is given according to the $1/f$ dependency as:

$$p_0(f) = p_0(f_0)/f \qquad (6)$$

With this extra $1/f$ condition we observe the second pitch evoking example (b):

2.b. A fundamental with 3 successive higher harmonics.

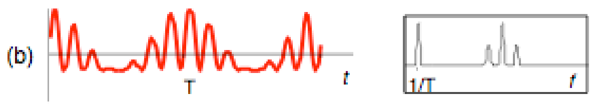

Fig. 2. Waveform and spectrum of a fundamental f_0 with the successive higher harmonics $5f_0$, $6f_0$ and $7f_0$[example b from De Cheveigné[3]]

This example has sinus frequency contributions of f_0 , $5f_0$, $6f_0$ and $7f_0$ with the chosen $1/f$ sound pressure amplitude ratio of $1 \div 1/5 \div 1/6 \div 1/7$, according to:

$$p(f_i) = \frac{p_0}{f_i}\sin(2\pi f_i t) \quad \text{with i = 1; 5; 6; 7} \qquad (7)$$

Including the transfer factor A for perilymph movements and after differentiation the perilymph velocity is given as:

$$v = v_0 \sum_{\mathrm{i}} \cos(2\pi f_{\mathrm{i}} t) = 2\pi A\, p_0 \sum_{\mathrm{i}} \cos(2\pi f_{\mathrm{i}} t) \qquad (8)$$

with i = 1; 5; 6; 7.

The squaring process by using the Bernoulli equation on this four terms Fourier series gives for each frequency f_{i} with i = 1; 5; 6; 7 the squared contribution:

$$\Delta p_{\mathrm{i}} = \Delta p_0 \left[\tfrac{1}{2} + \tfrac{1}{2}\cos\left(4\pi f_{\mathrm{i}} t\right)\right] \qquad (9)$$

So the four frequencies f_0 , $5f_0$, $6f_0$ and $7f_0$ evoke a combined static pressure:

$$\Delta p_{\mathrm{s}} = 2\Delta p_0 \qquad (10)$$

And four frequency dependent contributions with equal amplitude $\frac{1}{2}\Delta p_0$ and frequencies respectively $2f_0$; $10f_0$; $12f_0$ and $14f_0$.

But also all the combinations:

$$\Delta p_{\mathrm{i,j}} = 2\Delta p_0 \cos\left(2\pi f_{\mathrm{i}} t\right) \cos\left(2\pi f_{\mathrm{j}} t\right) \qquad (11)$$

With (i,j) = (1,5) (1,6) (1,7) (5,6) (6,7) (5,7)

Each of these 6 combinations can be written as:

$$\Delta p_{\mathrm{i,j}} = \Delta p_0 \{\cos\left[2\pi(f_{\mathrm{i}} - f_{\mathrm{j}})t\right] + \cos\left[2\pi(f_{\mathrm{i}} + f_{\mathrm{j}})t\right]\} \qquad (12)$$

This means that next to the already calculated doubled frequency contributions the following difference frequency contributions f_0 (twice); $2f_0$; $4f_0$; $5f_0$ and $6f_0$, all with equal amplitudes Δp_0.

And the sum frequencies: $6f_0$; $7f_0$; $8f_0$; $11f_0$; $12f_0$ and $13f_0$ again with equal amplitudes Δp_0.

So in the signal evoked on the basilar membrane we can distinguish the frequency spectrum in Table 1:

Table 1. Offered $1/f$ sound pressure spectrum f_0 , $5f_0$, $6f_0$ and $7f_0$ resulting in the corresponding sound energy spectrum evoked on the basilar membrane.

<table>
<tr><td colspan="12">Sound pressure frequencies on eardrum × f_0</td></tr>
<tr><td colspan="3">1</td><td colspan="3">5</td><td colspan="3">6</td><td colspan="3">7</td></tr>
<tr><td colspan="12">Corresponding amplitude × p_0</td></tr>
<tr><td colspan="3">1</td><td colspan="3">1 / 5</td><td colspan="3">1 / 6</td><td colspan="3">1 / 7</td></tr>
<tr><td colspan="12">Frequency contribution × f_0 on basilar membrane</td></tr>
<tr><td>1</td><td>2</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr>
<tr><td colspan="12">Corresponding stimulus amplitude × $\frac{1}{2}\Delta p_0$</td></tr>
<tr><td>4</td><td>3</td><td>2</td><td>2</td><td>4</td><td>2</td><td>2</td><td>1</td><td>2</td><td>3</td><td>2</td><td>1</td></tr>
</table>

Here we see two contributions respectively $F0$ and $6F0$ with a relative amplitude of 4. So here it is clear that the lowest of these two, being $F0$, is the pitch frequency.

The plots of these two frequency spectra are given in the next figures.

And now we see another effect: This $F0$ equal to the frequency f_0 is the stimulus on the basilar membrane, but if we look at example (a), the frequency of the stimulus on the basilar membrane in that case $F0$ is equal to the frequency $2f_0$. This because the only frequency contribution f_0 in (a) doubles due to the squaring there, while in (b) the $F0$ is formed by the contributions of the difference frequencies in the triplet formed by $5f_0$; $6f_0$ and $7f_0$.

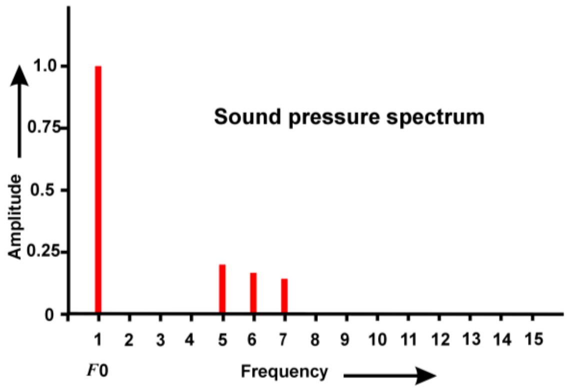

Fig. 3. Sound pressure spectrum of a fundamental f_0 with the successive higher harmonics $5f_0$, $6f_0$ and $7f_0$ and $1/f$ amplitude ratio.

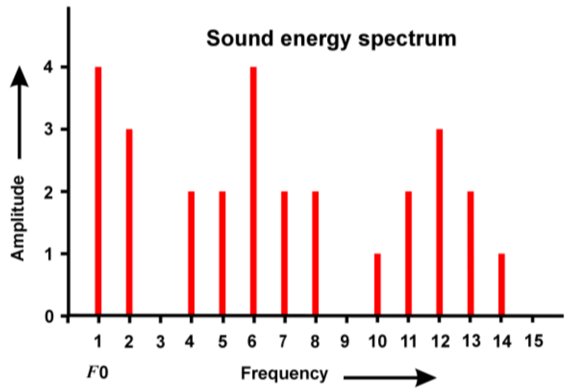

Fig. 4. Sound energy spectrum corresponding to a fundamental f_0 with the successive higher harmonics $5f_0$, $6f_0$ and $7f_0$ and $1/f$ amplitude ratio.

This one octave difference in pitch between the examples (a) and (b) is identical to the phenomenon that the strike note of a bell is actually perceived to be one octave lower than the expected pitch or fundamental frequency of that bell. A mysterious phenomenon already observed by Rayleigh[7], investigated by Jones[8] and later thought to be explained by Schouten et al[9], but now explained within the new paradigm as the fundamental different behavior in the transfer from sound pressure stimulus to sound energy stimulus of a single pure tone versus that of a harmonic tone complex.

The effect that the pitch frequency of a pure tone is twice as high as the pitch frequency of a harmonic series existing of all sine or cosine contributions can be heard in sound experiments, described by Heerens and De Ru[1].

2.c. Three successive higher harmonics with a missing fundamental.

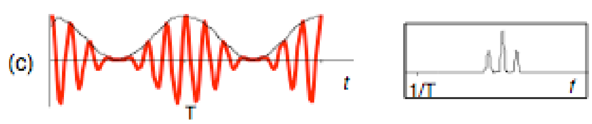

Fig. 5. Waveform and spectrum of three successive higher harmonics $5f_0$, $6f_0$ and $7f_0$ with a missing fundamental f_0 [example c from De Cheveigné[3]]

This example has sinus frequency contributions of $5f_0$, $6f_0$ and $7f_0$ with the chosen $1/f$ sound pressure amplitude ratio of $1/5 \div 1/6 \div 1/7$, according to:

$$p(f_\mathrm{i}) = \frac{p_0}{f_\mathrm{i}}\sin(2\pi f_\mathrm{i} t) \quad \text{with i = 5; 6; 7} \tag{13}$$

The assumed pitch equal to f_0 is clearly missing. Again, including the transfer factor A for perilymph movements and after differentiation the perilymph velocity is given as:

$$v = v_0 \sum_\mathrm{i} \cos(2\pi f_\mathrm{i} t) = 2\pi A\, p_0 \sum_\mathrm{i} \cos(2\pi f_\mathrm{i} t) \tag{14}$$

with i = 5; 6; 7.

The squaring process by using the Bernoulli equation on this four terms Fourier series gives for each frequency f_i with i = 5; 6; 7 the squared contribution:

$$\Delta p_\mathrm{i} = \Delta p_0 \left[\tfrac{1}{2} + \tfrac{1}{2}\cos(4\pi f_\mathrm{i} t)\right] \tag{15}$$

So now the three frequencies $5f_0$, $6f_0$ and $7f_0$ evoke a combined static pressure:

$$\Delta p_s = \tfrac{3}{2}\Delta p_0 \tag{16}$$

And three frequency dependent contributions with equal amplitude $\frac{1}{2}\Delta p_0$ and frequencies respectively $10f_0$; $12f_0$ and $14f_0$.

But also all the combinations:

$$\Delta p_{\mathrm{i,j}} = 2\Delta p_0 \cos(2\pi f_\mathrm{i} t)\cos(2\pi f_\mathrm{j} t) \tag{17}$$

With (i, j) = (5,6) (6,7) (5,7)

Again each of these 3 combinations can be written as:

$$\Delta p_{\mathrm{i,j}} = \Delta p_0\{\cos[2\pi(f_\mathrm{i} - f_\mathrm{j})t] + \cos[2\pi(f_\mathrm{i} + f_\mathrm{j})t]\} \tag{18}$$

This means that next to the already calculated doubled frequency contributions the following difference frequency contributions f_0 (twice) and $2f_0$ both with equal amplitudes Δp_0.

And the sum frequencies: $11f_0$; $12f_0$ and $13f_0$ again with equal amplitudes Δp_0.

So in the signal evoked on the basilar membrane we can distinguish the frequency spectrum in Table 2.

Table 2. Offered $1/f$ triplet sound pressure spectrum $5f_0$, $6f_0$ and $7f_0$ resulting in the corresponding sound energy spectrum evoked on the basilar membrane.

Sound pressure frequencies on eardrum × f_0						
5		6		7		
Corresponding amplitude × p_0						
1 / 5		1 / 6		1 / 7		
Frequency contribution × f_0 on basilar membrane						
1	2	10	11	12	13	14
Corresponding stimulus amplitude × $\frac{1}{2}\Delta p_0$						
4	2	1	2	3	2	1

The plots of these two frequency spectra are given in the next two figures.

Although the fundamental f_0 that is characteristic for this triplet is missed, as is shown in Fig. 6. it appears as the strongest contribution in the total energy frequency spectrum, shown in Fig. 7.

Now we can make an important remark:

The calculations with equations (13) – (18) and especially in the frequency difference contribution calculations the differences $f_\mathrm{i} - f_\mathrm{j}$ for each combination of i and j remain integer multiples of f_0 .

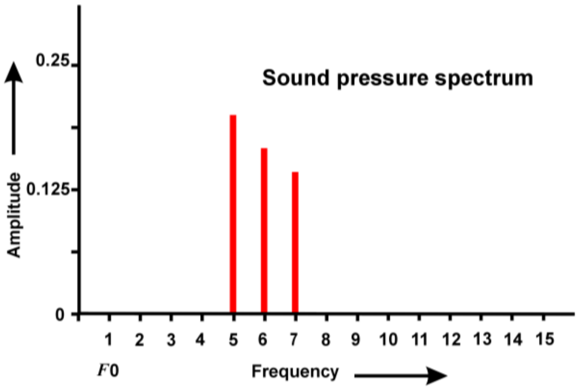

Fig. 6. Sound pressure spectrum of a triplet with successive higher harmonics $5f_0$, $6f_0$ and $7f_0$ and $1/f$ amplitude ratio, of which the fundamental f_0 is missed.

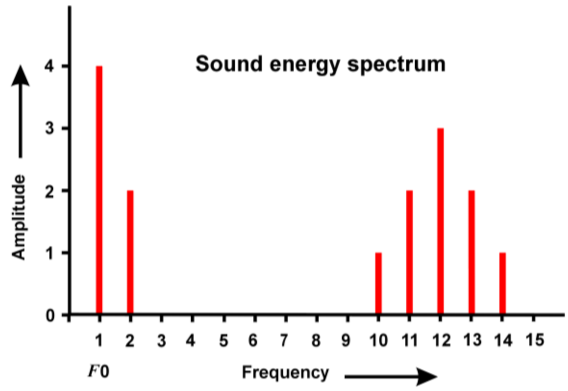

Fig. 7. Sound energy spectrum corresponding to a triplet with successive higher harmonics $5f_0$, $6f_0$ and $7f_0$ and $1/f$ amplitude ratio. Difference frequency f_0 is the strongest contribution on the basilar membrane

So as long as the $1/f$ sound pressure amplitude ratio is maintained, an arbitrary shift of the entire triplet – which even can be any fraction or multiply of the triplet frequency interval $\Delta f = f_0$ – to lower or higher frequencies, the amplitudes of both the first fundamental f_0 and the second fundamental $2f_0$ do not change. Neither do their frequencies.

This is also the result of the following experiment, described by Heerens and De Ru[1], where in contradiction to the experiments of De Boer[10] combined beat and frequency shift experiments clearly show no corresponding pitch shift.

For example:

The experiment starts with the tone complex built up by the higher harmonics of a missing fundamental of 200 Hz:

1400+1600+1800+2000+2200+2400+2600 Hz (A)

Then all frequencies will be shifted over 30 Hz, which forms the series:

1430+1630+1830+2030+2230+2430+2630 Hz (B)

According to the experiments described by De Boer the pitch of this shifted complex with a center frequency of 2030 Hz and difference frequency of 200 Hz can by good approximation be perceived to be equal to the sub harmonic frequency of the center frequency 2030 Hz of the entire complex, which is closest to the difference frequency of 200 Hz of the shifted tone complex. Hence one tenth of the 2030 Hz of the center frequency, which leads to 203 Hz.

So the enharmonic tone complex of (B) can best be compared to that of the harmonic tone complex of (C):

$$1421+1624+1827+2030+2233+2436+2639 \text{ Hz.} \qquad \text{(C)}$$

However an addition of a twin tone 203+406 Hz to the tone complex of (B) with a relative amplitude of 6, instead of adding an extra intensity to the pitch, a 3 Hz beat is clearly heard. When a 1 Hz lower twin tone, 202+404 Hz, with the same relative amplitude of 6 is used one hears a beat of 2 Hz, and in case of a twin tone of 201+402 Hz, a 1 Hz beat. The beat disappeared completely for the twin tone of 200+400 Hz and finally, in case of the twin tone of 199 + 398 Hz the beat returned to 1 Hz.

The same experiment with the 203+406 Hz twin tone addition to the (C) complex shows that there doesn't exist any beat phenomenon, but addition of both 202+404 Hz or 204+408 Hz to the complex of (B) evokes a clear beat of 1 Hz.

The only possible conclusion out of these results is that an equal shift of all frequency contributions in a multiple tone combination does not lead to a pitch shift corresponding to the sub-harmonic of the nearest harmonic complex.

The subjective interpretations of the well trained experimental subjects in the experiments of De Boer are nothing more than an illusion.

2.d. Three successive higher harmonics with frequency spacing twice the frequency of the missing fundamental.

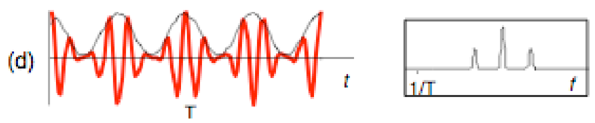

Fig. 8. Waveform and spectrum of three successive higher harmonics $5f_0$, $7f_0$ and $9f_0$ with a missing fundamental f_0 [example d from De Cheveigné[3]]

Again, including the transfer factor A for perilymph movements and after differentiation the perilymph velocity is given as:

$$v = v_0 \sum_i \cos(2\pi f_i t) = 2\pi A\, p_0 \sum_i \cos(2\pi f_i t) \quad (20)$$

but now with i = 5; 7; 9.

The squaring process by using the Bernoulli equation on this four terms Fourier series gives for each frequency f_i now with i = 5; 7; 9 the squared contribution:

$$\Delta p_i = \Delta p_0 \left[\tfrac{1}{2} + \tfrac{1}{2}\cos(4\pi f_i t)\right] \quad (21)$$

So now the three frequencies $5f_0$, $6f_0$ and $7f_0$ evoke a combined static pressure:

$$\Delta p_s = \tfrac{3}{2}\Delta p_0 \quad (22)$$

Which is identical to the example in paragraph 2.c. and three frequency dependent contributions with equal amplitude $\tfrac{1}{2}\Delta p_0$ and frequencies respectively $10f_0$; $14f_0$ and $18f_0$.

But also all the combinations:

$$\Delta p_{i,j} = 2\Delta p_0 \cos(2\pi f_i t)\cos(2\pi f_j t) \quad (23)$$

With (i, j) = (5,7) (7,9) (5,9)

Again each of these 3 combinations can be written as:

$$\Delta p_{i,j} = \Delta p_0\{\cos[2\pi(f_i - f_j)t] + \cos[2\pi(f_i + f_j)t]\} \quad (24)$$

This means that next to the already calculated doubled frequency contributions the following difference frequency contributions $2f_0$ (twice) and $4f_0$ both with equal amplitudes Δp_0. But now the fundamental f_0, belonging to the series of $5f_0$, $6f_0$ and $7f_0$ is missing.

And the sum frequencies: $12f_0$; $14f_0$ and $16f_0$ again with equal amplitudes Δp_0.

So in the signal evoked on the basilar membrane we can distinguish the frequency spectrum in Table 3.

Table 3. Offered $1/f$ triplet sound pressure spectrum $5f_0$, $7f_0$ and $9f_0$ resulting in the corresponding sound energy spectrum evoked on the basilar membrane.

Sound pressure frequencies on eardrum × f_0							
5			7			9	
Corresponding amplitude × p_0							
1 / 5			1 / 7			1 / 9	
Frequency contribution × f_0 on basilar membrane							
1	2	4	10	12	14	16	18
Corresponding stimulus amplitude × $\tfrac{1}{2}\Delta p_0$							
0	4	2	1	2	3	2	1

The plots of these two frequency spectra are given in the next two figures.

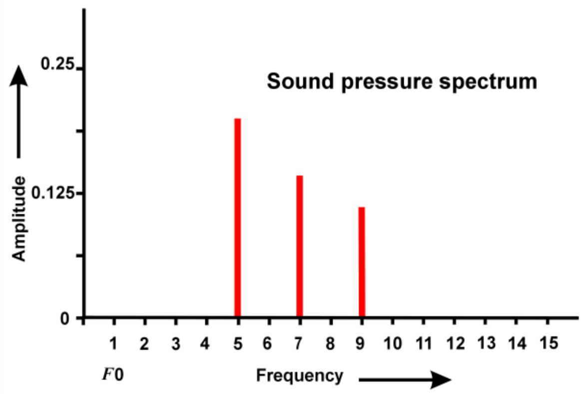

Fig. 9. Sound pressure spectrum of a triplet with successive higher harmonics $5f_0$, $7f_0$ and $9f_0$ and $1/f$ amplitude ratio, of which the fundamental f_0 is missed.

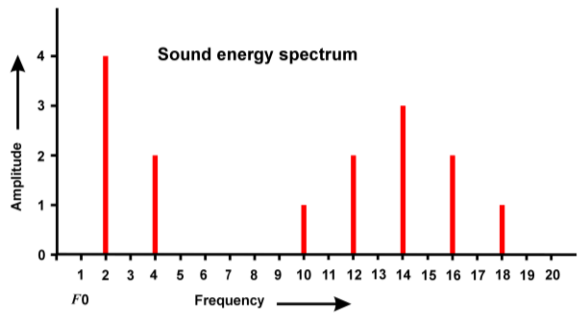

Fig. 10. Sound energy spectrum corresponding to a triplet with successive higher harmonics $5f_0$, $7f_0$ and $9f_0$ and $1/f$ amplitude ratio. Difference frequency f_0 is missed and $2f_0$ is the strongest contribution on the basilar membrane.

We now can observe this energy frequency spectrum as the result of a sound pressure triplet with spacing Δf between successive contributions equal to $2f_0$, which his shifted over

$f_0 = \frac{1}{2}\Delta f$ upwards or downwards from the ideal harmonic complex, where each frequency contribution is an integer multiple of the fundamental $2f_0$.

But we have seen in paragraph 2.c that such a frequency shift does not influence both the frequency and amplitude of the pitch, which is based on the frequency difference $\Delta f = 2f_0$ in the triplet.

And therefore this sound pressure complex is heard with a pitch twice as high as in example 2.c. Despite the shift over f_0.

2.e. Nine successive harmonics, starting with the fundamental but alternatively sine and cosine contributions.

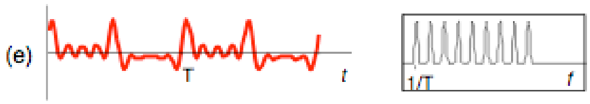

Fig. 11. Waveform and spectrum of nine successive harmonics $f_0 - 9f_0$ with alternating sine and cosine contributions [example e from De Cheveigné[3]]

This example has sinus frequency contributions of f_0, $3f_0$, $5f_0$, $7f_0$ and $9f_0$, but cosine contributions of $2f_0$, $4f_0$, $6f_0$ and $8f_0$. All with the chosen $1/f$ sound pressure amplitude ratio, according to:

$$p(f_\mathrm{i}) = \frac{p_0}{f_\mathrm{i}}\sin(2\pi f_\mathrm{i} t) \tag{25}$$

with i = 1; 3; 5; 7 and 9.

The other contributions are:

$$p(f_\mathrm{i}) = \frac{p_0}{f_\mathrm{i}}\cos(2\pi f_\mathrm{i} t) \tag{26}$$

with i = 2; 4; 6; and 8.

Including the transfer factor A for perilymph movements and after differentiation the perilymph velocity is given as:

$$v = v_0 \textstyle\sum_\mathrm{i} \cos(2\pi f_\mathrm{i} t) = 2\pi A\, p_0 \sum_\mathrm{i} \cos(2\pi f_\mathrm{i} t) \tag{27}$$

with i = 1; 3; 5; 7 and 9,

combined with

$$v = -v_0 \textstyle\sum_\mathrm{i} \sin(2\pi f_\mathrm{i} t) = -2\pi A\, p_0 \sum_\mathrm{i} \sin(2\pi f_\mathrm{i} t) \tag{28}$$

with i = 2; 4; 6; and 8.

It leads to a rather extended sound energy frequency spectrum given in Fig. 13.

The squaring process by using the Bernoulli equation on all the five terms with i = 1, 3, 5, 7 and 9 results in contribution on the basilar membrane:

$$\Delta p_\mathrm{i} = \frac{1}{2}\Delta p_0[1 + \cos(4\pi f_\mathrm{i} t)] \tag{29}$$

While the terms with j = 2; 4; 6; and 8 result in the contributions:

$$\Delta p_\mathrm{j} = \frac{1}{2}\Delta p_0\left[1 - \cos\left(4\pi f_\mathrm{j} t\right)\right] \tag{30}$$

So the nine frequencies f_0 - $9f_0$ evoke a combined static pressure on the basilar membrane:

$$\Delta p_\mathrm{s} = \frac{9}{2}\Delta p_0 \tag{31}$$

The frequency contributions of $2f_0$, $6f_0$, $10f_0$, $14f_0$ and $18f_0$ evoke $\cos(4\pi f_\mathrm{i} t)$, while $4f_0$, $8f_0$, $12f_0$, and $16f_0$ evoke $-\cos\left(4\pi f_\mathrm{j} t\right)$. All with equal amplitude $\frac{1}{2}\Delta p_0$.

But also all the following combinations are evoked:

$$\Delta p_\mathrm{i,j} = 2\Delta p_0 \cos(2\pi f_\mathrm{i} t)\cos\left(2\pi f_\mathrm{j} t\right) \tag{32}$$

or:

$$\Delta p_\mathrm{i,j} = \Delta p_0\left\{\cos\left[2\pi\left(f_\mathrm{i} - f_\mathrm{j}\right)t\right] + \cos\left[2\pi\left(f_\mathrm{i} + f_\mathrm{j}\right)t\right]\right\} \tag{33}$$

with:
$(\mathrm{i,j}) = (1{,}3)(1{,}5)(1{,}7)(1{,}9)(3{,}5)(3{,}7)(3{,}9)(5{,}7)(5{,}9)(7{,}9)$

The combinations:

$$\Delta p_\mathrm{i,j} = 2\Delta p_0 \sin(2\pi f_\mathrm{i} t)\sin\left(2\pi f_\mathrm{j} t\right) \tag{34}$$

or:

$$\Delta p_\mathrm{i,j} = \Delta p_0\left\{\cos\left[2\pi\left(f_\mathrm{i} - f_\mathrm{j}\right)t\right] + \cos\left[2\pi\left(f_\mathrm{i} + f_\mathrm{j}\right)t\right]\right\} \tag{35}$$

with:
$(\mathrm{i,j}) = (2{,}4)(2{,}6)(2{,}8)(4{,}6)(4{,}8)(6{,}8)$

The combinations:

$$\Delta p_\mathrm{i,j} = -2\Delta p_0 \cos(2\pi f_\mathrm{i} t)\sin\left(2\pi f_\mathrm{j} t\right) \tag{36}$$

or:

$$\Delta p_\mathrm{i,j} = \Delta p_0\left\{\sin\left[2\pi\left(f_\mathrm{i} - f_\mathrm{j}\right)t\right] - \sin\left[2\pi\left(f_\mathrm{i} + f_\mathrm{j}\right)t\right]\right\} \tag{37}$$

with:
$(\mathrm{i,j}) = (1{,}2)(1{,}4)(1{,}6)(1{,}8)(3{,}4)(3{,}6)(3{,}8)(5{,}6)(5{,}8)(7{,}8)$

And the combinations:

$$\Delta p_\mathrm{i,j} = -2\Delta p_0 \sin(2\pi f_\mathrm{i} t)\cos\left(2\pi f_\mathrm{j} t\right) \tag{38}$$

or:

$$\Delta p_\mathrm{i,j} = -\Delta p_0\left\{\sin\left[2\pi\left(f_\mathrm{i} - f_\mathrm{j}\right)t\right] + \sin\left[2\pi\left(f_\mathrm{i} + f_\mathrm{j}\right)t\right]\right\} \tag{39}$$

with:
$(\mathrm{i,j}) = (2{,}3)(2{,}5)(2{,}7)(2{,}9)(4{,}5)(4{,}7)(4{,}9)(6{,}7)(6{,}9)(8{,}9)$

So after all the individual calculations and grouping of identical contributions, the signal evoked on the basilar membrane we can distinguish the frequency spectrum of Table 4:

Table 4. Offered $1/f$ sound pressure spectrum f_0 to $9f_0$ with alternating sine and cosine contributions resulting in the corresponding sound energy spectrum evoked on the basilar membrane.

Sound pressure frequencies on eardrum × f_0								
1	2	3	4	5	6	7	8	9
Corresponding amplitude × p_0								
1	1 / 2	1 / 3	1 / 4	1 / 5	1 / 6	1 / 7	1 / 8	1 / 9
Frequency contribution × f_0 on basilar membrane								
1	2	3	4	5	6	7	8	9
Corresponding stimulus amplitude × $\frac{1}{2}\Delta p_0$								
-sin 0	cos 15	-sin 2	cos 13	-sin 4	cos 11	-sin 6	cos 7	-sin 8
Frequency contribution × f_0 on basilar membrane								
10	11	12	13	14	15	16	17	18
Corresponding stimulus amplitude × $\frac{1}{2}\Delta p_0$								
cos 9	-sin 8	cos 5	-sin 6	cos 5	-sin 4	cos 1	-sin 2	cos 1

For the fundamental $\boldsymbol{F0} = f_0$ a peculiar condition exists:

The contributions of $\Delta p_0 \sin\left[2\pi\left(f_\mathrm{i} - f_\mathrm{j}\right)t\right]$ in Eq.(37) for the four combinations $(\mathrm{i,j}) = (1{,}2)(3{,}4)(5{,}6)(7{,}8)$ and the contributions $-\Delta p_0 \sin\left[2\pi\left(f_\mathrm{i} - f_\mathrm{j}\right)t\right]$ in Eq.(39) for the combinations $(\mathrm{i,j}) = (2{,}3)(4{,}5)(6{,}7)(8{,}9)$ are all forming contributions to the f_0 peak, but pair by pair with opposite sign, so they all cancel, resulting in a missing $\boldsymbol{F0}$.

The other remarkable effect is given in Table 4.:

All the odd harmonics have total contributions in the form of:

$$\Delta p_{\mathrm{i}} = -A_{t\,\mathrm{i}}\,\tfrac{1}{2}\Delta p_0 \sin\left(2\pi f_{\mathrm{i}} t\right) \tag{40}$$

While all the even harmonics have total contributions in the form of:

$$\Delta p_{\mathrm{i}} = A_{t\,\mathrm{i}}\,\tfrac{1}{2}\Delta p_0 \cos\left(2\pi f_{\mathrm{i}} t\right) \tag{41}$$

Where $A_{t\,\mathrm{i}}$ is the corresponding stimulus amplitude, given in Table 4.

The sound pressure frequency spectrum is given in Fig. 12, where the blue peaks are indicating the sine and the red peaks the cosine contributions.

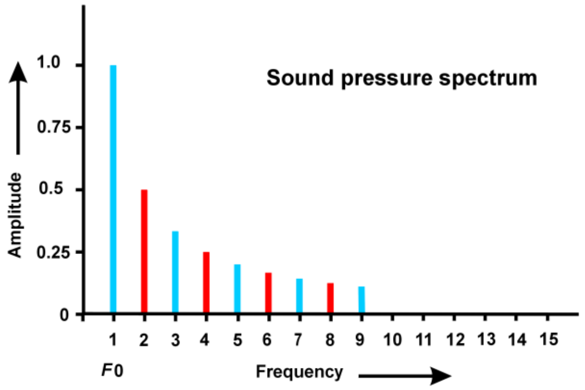

Fig. 12. Sound pressure spectrum corresponding to the complex of harmonics f_0 to $9f_0$ and $1/f$ amplitude ratio. Odd harmonics [blue] are sine functions, even harmonics [red] are cosine functions.

The sound energy frequency spectrum is given in Fig. 13. where the blue peaks are sine functions with a $180°$ phase shift related to the incoming odd sound pressure frequency contributions.

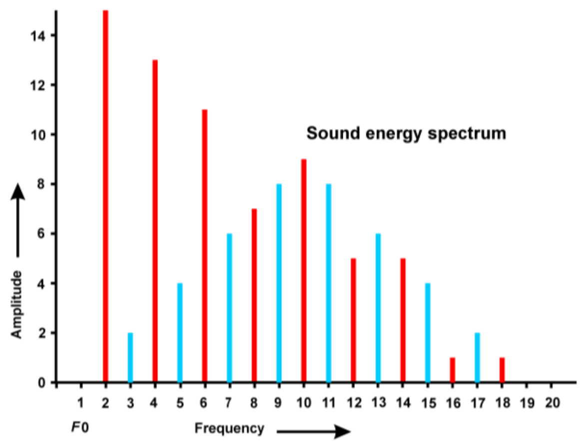

Fig. 13. Sound energy spectrum corresponding to the complex of harmonics f_0 to $9f_0$ and $1/f$ amplitude ratio. Odd harmonics [blue] are now – sine functions, even harmonics [red] are cosine functions.

In this spectrum it is clear that the fundamental $\boldsymbol{F0} = f_0$ is missed. The second in row: $\boldsymbol{2F0}$ is the strongest contribution and will therefore act as pitch representing frequency.

In this example the pitch is heard one octave higher than can be expected from the setup of the harmonic composition.

2.f. Nine successive harmonics, starting with the fundamental while all contributions are cosine functions.

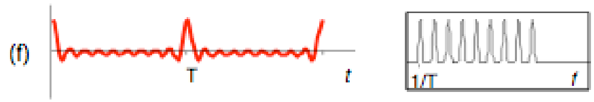

Fig. 14. Waveform and spectrum of nine successive harmonics f_0 – $9f_0$ with cosine contributions [example e from De Cheveigné[3]]

This example has nine cosine frequency contributions starting with f_0 and ending with $9f_0$. All with the chosen $1/f$ sound pressure amplitude ratio, according to:

$$p(f_{\mathrm{i}}) = \frac{p_0}{f_{\mathrm{i}}}\cos(2\pi f_{\mathrm{i}} t) \tag{42}$$

with i = 1 – 9.

Including the transfer factor A for perilymph movements and after differentiation the perilymph velocity is given as:

$$v = v_0 \textstyle\sum_{\mathrm{i}} \sin(2\pi f_{\mathrm{i}} t) = -2\pi A\, p_0 \sum_{\mathrm{i}} \sin(2\pi f_{\mathrm{i}} t) \tag{43}$$

with i = 1 – 9.

It also leads to a rather extended sound energy frequency spectrum given in Fig. 13.

The squaring process by using the Bernoulli equation on all the nine terms with i = 1 – 9 results in contribution on the basilar membrane:

$$\Delta p_{\mathrm{i}} = \frac{1}{2}\Delta p_0 [1 - \cos\left(4\pi f_{\mathrm{i}} t\right)] \tag{44}$$

So the nine frequencies f_0 - $9f_0$ evoke like in paragraph 2.e a combined static pressure on the basilar membrane:

$$\Delta p_s = \tfrac{9}{2}\Delta p_0 \tag{45}$$

And evoke frequency contributions $-\cos\left(4\pi f_{\mathrm{i}} t\right)$, with f_{i} equal to respectively $2f_0$, $4f_0$, $6f_0$, $8f_0$, $10f_0$, $12f_0$, $14f_0$, $16f_0$ and $18f_0$. All with equal amplitude $\frac{1}{2}\Delta p_0$.

But also all the following combinations are evoked:

$$\Delta p_{\mathrm{i,j}} = 2\Delta p_0 \sin\left(2\pi f_{\mathrm{i}} t\right) \sin\left(2\pi f_{\mathrm{j}} t\right) \tag{46}$$

or:

$$\Delta p_{\mathrm{i,j}} = \Delta p_0 \{\cos\left[2\pi\left(f_{\mathrm{i}} - f_{\mathrm{j}}\right)t\right] - \cos\left[2\pi\left(f_{\mathrm{i}} + f_{\mathrm{j}}\right)t\right]\} \tag{47}$$

with:
(i,j) = (1,2)(1,3)(1,4)(1,5)(1,6)(1,7)(1,8)(1,9)
(2,3)(2,4)(2,5)(2,6)(2,7)(2,8)(2,9)
(3,4)(3,5)(3,6)(3,7)(3,8)(3,9) (4,5)(4,6)(4,7)(4,8)(4,9)
(5,6)(5,7)(5,8)(5,9) (6,7)(6,8)(6,9) (7,8)(7,9) (8,9)

So after all the individual calculations and grouping of identical contributions, the signal evoked on the basilar membrane we can distinguish the frequency spectrum of Table 4.

The fundamental $\boldsymbol{F0} = f_0$ is the strongest in this series.

All the harmonics have total contributions in the form of:

$$\Delta p_{\mathrm{i}} = A_{t\,\mathrm{i}}\,\tfrac{1}{2}\Delta p_0 \cos\left(2\pi f_{\mathrm{i}} t\right) \tag{48}$$

Where $A_{t\,\mathrm{i}}$ is the corresponding stimulus amplitude, given in Table 5.

The sound pressure frequency spectrum is given in Fig. 15.

In Fig.16. the sound energy frequency spectrum is shown, where the red peaks indicate the positive cosine and the blue peaks are indicating the negative cosine contributions or 180° phase shifted contributions.

Table 5. Offered $1/f$ sound pressure spectrum f_0 to $9f_0$ with all cosine contributions resulting in the corresponding sound energy spectrum evoked on the basilar membrane.

Sound pressure frequencies on eardrum $\times f_0$								
1	2	3	4	5	6	7	8	9
Corresponding amplitude $\times p_0$								
1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9
Frequency contribution $\times f_0$ on basilar membrane								
1	2	3	4	5	6	7	8	9
Corresponding stimulus amplitude $\times \frac{1}{2}\Delta p_0$								
cos 16	cos 13	cos 10	cos 9	cos 4	cos 1	-cos 2	-cos 5	-cos 8
Frequency contribution $\times f_0$ on basilar membrane								
10	11	12	13	14	15	16	17	18
Corresponding stimulus amplitude $\times \frac{1}{2}\Delta p_0$								
-cos 9	-cos 8	-cos 7	-cos 6	-cos 5	-cos 4	-cos 3	-cos 2	-cos 1

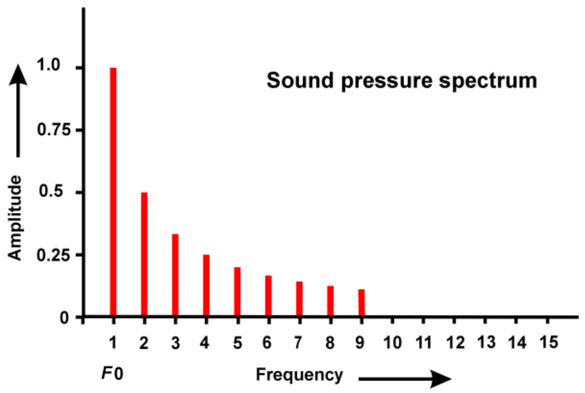

Fig. 15. Sound pressure spectrum corresponding to the complex of harmonics f_0 to $9f_0$ and $1/f$ amplitude ratio. All harmonics are cosine functions.

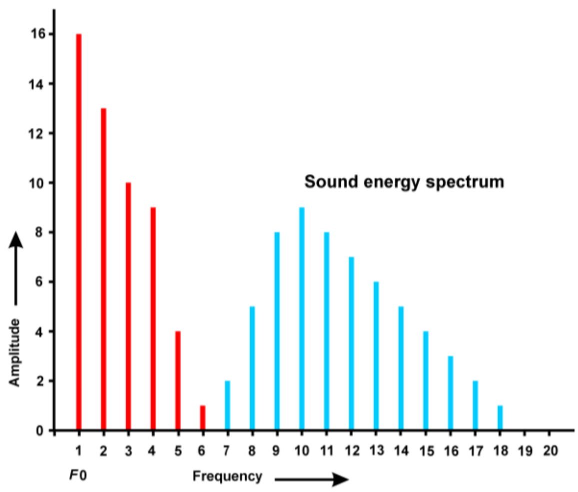

Fig. 16. Sound energy spectrum corresponding to the complex of harmonics f_0 to $9f_0$ with all cosine contributions and $1/f$ amplitude ratio. Lower harmonics [red] with positive sign and the higher harmonics [bleu] with opposite sign or 180° phase shift.

Apart from the phase shift that takes place between $6f_0$ and $7f_0$we observe a strong dip in the spectrum around the same location on the frequency scale.

This can also be seen in Fig. 14. There is shown a strong f_0 contribution in the form of a high peak with a repetition period of T while the rest of the waveform is dominated by higher frequency contributions.

2.g. Nine successive harmonics, starting with the fundamental while all contributions are sine functions as comparison.

In order to overview the influence of the sine and cosine choices in the contributions we can use all sine instead of all cosine contributions.

This example has nine cosine frequency contributions starting with f_0 and ending with $9f_0$. All with the chosen $1/f$ sound pressure amplitude ratio, according to:

$$p(f_i) = \frac{p_0}{f_i}\sin(2\pi f_i t) \qquad (49)$$

with i = 1 – 9.

Including the transfer factor A for perilymph movements and after differentiation the perilymph velocity is given as:

$$v = v_0 \sum_i \cos(2\pi f_i t) = 2\pi A\, p_0 \sum_i \cos(2\pi f_i t) \qquad (50)$$

with i = 1 – 9.

The squaring process by using the Bernoulli equation on all the nine terms with i = 1 – 9 results in contribution on the basilar membrane:

$$\Delta p_i = \frac{1}{2}\Delta p_0[1 + \cos(4\pi f_i t)] \qquad (51)$$

So the nine frequencies f_0 - $9f_0$ evoke like in paragraph 2.e a combined static pressure on the basilar membrane:

$$\Delta p_s = \frac{9}{2}\Delta p_0 \qquad (52)$$

And evoke frequency contributions $\cos(4\pi f_i t)$, with f_i equal to respectively $2f_0$, $4f_0$, $6f_0$, $8f_0$, $10f_0$, $12f_0$, $14f_0$, $16f_0$ and $18f_0$. All with equal amplitude $\frac{1}{2}\Delta p_0$.

But also all the following combinations are evoked:

$$\Delta p_{i,j} = 2\Delta p_0 \cos(2\pi f_i t)\cos(2\pi f_j t) \qquad (53)$$

or:

$$\Delta p_{i,j} = \Delta p_0\{\cos[2\pi(f_i - f_j)t] + \cos[2\pi(f_i + f_j)t]\} \qquad (54)$$

with:
$(i,j) = (1,2)(1,3)(1,4)(1,5)(1,6)(1,7)(1,8)(1,9)$
$(2,3)(2,4)(2,5)(2,6)(2,7)(2,8)(2,9)$
$(3,4)(3,5)(3,6)(3,7)(3,8)(3,9)$ $(4,5)(4,6)(4,7)(4,8)(4,9)$
$(5,6)(5,7)(5,8)(5,9)$ $(6,7)(6,8)(6,9)$ $(7,8)(7,9)$ $(8,9)$

So after all the individual calculations and grouping of identical contributions, in the signal evoked on the basilar membrane we distinguish the frequency spectrum of Table 4.

The fundamental $\boldsymbol{F0} = f_0$ is the strongest in this series.

All the harmonics have total contributions in the form of:

$$\Delta p_i = A_{t\,i}\,\frac{1}{2}\Delta p_0 \cos(2\pi f_i t) \qquad (55)$$

Where $A_{t\,i}$ is the corresponding stimulus amplitude, given in Table 5.

The sound pressure frequency spectrum is given in Fig. 17. where all the contributions are sine functions.

Table 6. Offered $1/f$ sound pressure spectrum f_0 to $9f_0$ with all sine contributions resulting in the corresponding sound energy spectrum evoked on the basilar membrane.

Sound pressure frequencies on eardrum × f_0								
1	2	3	4	5	6	7	8	9
Corresponding amplitude × p_0								
1	1 / 2	1 / 3	1 / 4	1 / 5	1 / 6	1 / 7	1 / 8	1 / 9
Frequency contribution × f_0 on basilar membrane								
1	2	3	4	5	6	7	8	9
Corresponding stimulus amplitude × $\frac{1}{2}\Delta p_0$								
16	15	14	13	12	11	10	9	8
Frequency contribution × f_0 on basilar membrane								
10	11	12	13	14	15	16	17	18
Corresponding stimulus amplitude × $\frac{1}{2}\Delta p_0$								
9	8	7	6	5	4	3	2	1

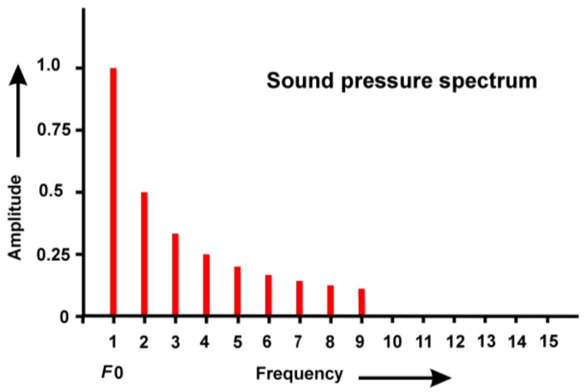

Fig. 17. Sound pressure spectrum corresponding to the complex of harmonics f_0 to $9f_0$ and $1/f$ amplitude ratio. All harmonics are sine functions.

In Fig.18. the sound energy frequency spectrum is shown, with a remarkable systematic structure.

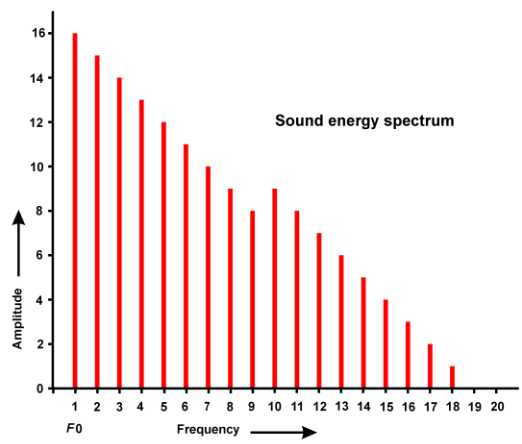

Fig. 18. Sound energy spectrum corresponding to the complex of harmonics f_0 to $9f_0$ with all cosine contributions and $1/f$ amplitude ratio. All these harmonic contributions have a zero phase shift.

2.h. Nine successive harmonics, starting with the fundamental but alternatively cosine and sine contributions.

This example has cosine frequency contributions of f_0 , $3f_0$, $5f_0$, $7f_0$ and $9f_0$, but sine contributions of $2f_0$, $4f_0$, $6f_0$ and $8f_0$. All with the chosen $1/f$ sound pressure amplitude ratio, according to:

$$p(f_{\mathrm{i}}) = \frac{p_0}{f_{\mathrm{i}}}\cos(2\pi f_{\mathrm{i}} t) \qquad (56)$$

with i = 1; 3; 5; 7 and 9.

The other contributions are:

$$p(f_{\mathrm{i}}) = \frac{p_0}{f_{\mathrm{i}}}\sin(2\pi f_{\mathrm{i}} t) \qquad (57)$$

with i = 2; 4; 6; and 8.

Including the transfer factor A for perilymph movements and after differentiation the perilymph velocity is given as:

$$v = -v_0 \textstyle\sum_{\mathrm{i}} \sin(2\pi f_{\mathrm{i}} t) = -2\pi A\, p_0 \sum_{\mathrm{i}} \sin(2\pi f_{\mathrm{i}} t) \qquad (58)$$

with i = 1; 3; 5; 7 and 9,

combined with

$$v = v_0 \textstyle\sum_{\mathrm{i}} \cos(2\pi f_{\mathrm{i}} t) = 2\pi A\, p_0 \sum_{\mathrm{i}} \cos(2\pi f_{\mathrm{i}} t) \qquad (59)$$

with i = 2; 4; 6; and 8.

The squaring process by using the Bernoulli equation on all the five terms with i = 1, 3, 5, 7 and 9 results in contribution on the basilar membrane:

$$\Delta p_{\mathrm{i}} = \frac{1}{2}\Delta p_0[1 - \cos(4\pi f_{\mathrm{i}} t)] \qquad (60)$$

While the terms with j = 2; 4; 6; and 8 result in the contributions:

$$\Delta p_{\mathrm{j}} = \frac{1}{2}\Delta p_0\left[1 + \cos\left(4\pi f_{\mathrm{j}} t\right)\right] \qquad (61)$$

So the nine frequencies f_0 - $9f_0$ evoke a combined static pressure on the basilar membrane:

$$\Delta p_s = \tfrac{9}{2}\Delta p_0 \qquad (62)$$

The frequency contributions of $2f_0$, $6f_0$, $10f_0$, $14f_0$ and $18f_0$ evoke $-\cos(4\pi f_{\mathrm{i}} t)$, while $4f_0$, $8f_0$, $12f_0$, and $16f_0$ evoke $\cos\left(4\pi f_{\mathrm{j}} t\right)$. All with equal amplitude $\frac{1}{2}\Delta p_0$.

But also all the following combinations are evoked:

$$\Delta p_{\mathrm{i,j}} = 2\Delta p_0 \sin(2\pi f_{\mathrm{i}} t) \sin\left(2\pi f_{\mathrm{j}} t\right) \qquad (63)$$

or:

$$\Delta p_{\mathrm{i,j}} = \Delta p_0\{\cos\left[2\pi(f_{\mathrm{i}} - f_{\mathrm{j}})t\right] - \cos\left[2\pi(f_{\mathrm{i}} + f_{\mathrm{j}})t\right]\} \qquad (33)$$

with:

(i,j) = (1,3)(1,5)(1,7)(1,9)(3,5)(3,7)(3,9)(5,7)(5,9)(7,9)

The combinations:

$$\Delta p_{\mathrm{i,j}} = 2\Delta p_0 \cos(2\pi f_{\mathrm{i}} t) \cos\left(2\pi f_{\mathrm{j}} t\right) \qquad (34)$$

or:

$$\Delta p_{\mathrm{i,j}} = \Delta p_0\{\cos\left[2\pi(f_{\mathrm{i}} - f_{\mathrm{j}})t\right] + \cos\left[2\pi(f_{\mathrm{i}} + f_{\mathrm{j}})t\right]\} \qquad (35)$$

with:

(i,j) = (2,4)(2,6)(2,8)(4,6)(4,8)(6,8)

The combinations:

$$\Delta p_{\mathrm{i,j}} = -2\Delta p_0 \sin(2\pi f_{\mathrm{i}} t) \cos\left(2\pi f_{\mathrm{j}} t\right) \qquad (36)$$

or:

$$\Delta p_{\mathrm{i,j}} = -\Delta p_0\{\sin\left[2\pi(f_{\mathrm{i}} - f_{\mathrm{j}})t\right] + \sin\left[2\pi(f_{\mathrm{i}} + f_{\mathrm{j}})t\right]\} \qquad (37)$$

with:

$(\mathrm{i,j}) = (1{,}2)(1{,}4)(1{,}6)(1{,}8)(3{,}4)(3{,}6)(3{,}8)(5{,}6)(5{,}8)(7{,}8)$

And the combinations:

$$\Delta p_{\mathrm{i,j}} = -2\Delta p_0 \cos\left(2\pi f_{\mathrm{i}} t\right) \sin\left(2\pi f_{\mathrm{j}} t\right) \qquad (38)$$

or:

$$\Delta p_{\mathrm{i,j}} = \Delta p_0 \{\sin\left[2\pi\left(f_{\mathrm{i}} - f_{\mathrm{j}}\right)t\right] - \sin\left[2\pi\left(f_{\mathrm{i}} + f_{\mathrm{j}}\right)t\right]\} \qquad (39)$$

with:

$(\mathrm{i,j}) = (2{,}3)(2{,}5)(2{,}7)(2{,}9)(4{,}5)(4{,}7)(4{,}9)(6{,}7)(6{,}9)(8{,}9)$

So after all the individual calculations and grouping of identical contributions, the signal evoked on the basilar membrane we can distinguish the frequency spectrum of Table 4:

Table 7. Offered $1/f$ sound pressure spectrum f_0 to $9f_0$ with alternating sine and cosine contributions resulting in the corresponding sound energy spectrum evoked on the basilar membrane.

Sound pressure frequencies on eardrum × f_0								
1	2	3	4	5	6	7	8	9
Corresponding amplitude × p_0								
1	1 / 2	1 / 3	1 / 4	1 / 5	1 / 6	1 / 7	1 / 8	1 / 9
Frequency contribution × f_0 on basilar membrane								
1	2	3	4	5	6	7	8	9
Corresponding stimulus amplitude × $\frac{1}{2}\Delta p_0$								
-sin 0	cos 13	-sin 2	cos 9	-sin 4	cos 5	-sin 6	cos 1	-sin 8
Frequency contribution × f_0 on basilar membrane								
10	11	12	13	14	15	16	17	18
Corresponding stimulus amplitude × $\frac{1}{2}\Delta p_0$								
-cos 1	-sin 8	-cos 1	-sin 6	-cos 1	-sin 4	-cos 1	-sin 2	-cos 1

For the fundamental $\boldsymbol{F0} = f_0$ a peculiar condition exists:

The contributions of $\Delta p_0 \sin\left[2\pi\left(f_{\mathrm{i}} - f_{\mathrm{j}}\right)t\right]$ in Eq.(37) for the four combinations $(\mathrm{i,j}) = (1{,}2)(3{,}4)(5{,}6)(7{,}8)$ and the contributions $-\Delta p_0 \sin\left[2\pi\left(f_{\mathrm{i}} - f_{\mathrm{j}}\right)t\right]$ in Eq.(39) for the combinations $(\mathrm{i,j}) = (2{,}3)(4{,}5)(6{,}7)(8{,}9)$ are all forming contributions to the f_0 peak, but pair by pair with opposite sign, so they all cancel, resulting in a missing $\boldsymbol{F0}$.

The other remarkable effect is given in Table 7.

All the odd harmonics have total contributions in the form of:

$$\Delta p_{\mathrm{i}} = -A_{t\,\mathrm{i}} \tfrac{1}{2} \Delta p_0 \sin\left(2\pi f_{\mathrm{i}} t\right) \qquad (40)$$

While all the even harmonics have total contributions in the form of:

$$\Delta p_{\mathrm{i}} = A_{t\,\mathrm{i}} \tfrac{1}{2} \Delta p_0 \cos\left(2\pi f_{\mathrm{i}} t\right) \qquad (41)$$

Where $A_{t\,\mathrm{i}}$ is the corresponding stimulus amplitude, given in Table 4. From $10 f_0$ all cosine contributions become negative or have a phase shift of $180°$.

The sound pressure frequency spectrum is given in Fig. 19, where the blue peaks are indicating the cosine and the red peaks the sine contributions.

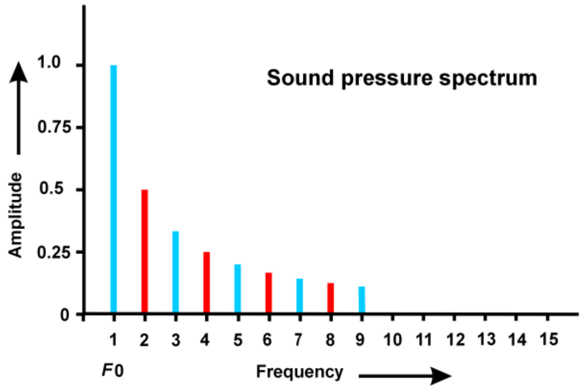

Fig. 19. Sound pressure spectrum corresponding to the complex of harmonics f_0 to $9f_0$ and $1/f$ amplitude ratio. Odd harmonics [blue] are cosine functions, even harmonics [red] are sine functions.

The sound energy frequency spectrum is given in Fig. 13. where the blue peaks are sine functions with a $180°$ phase shift related to the incoming odd sound pressure frequency contributions. The red peaks are even cosine functions, while the green ones are – cosine functions or cosine functions with a $180°$ phase shift related to the lower red frequency peaks.

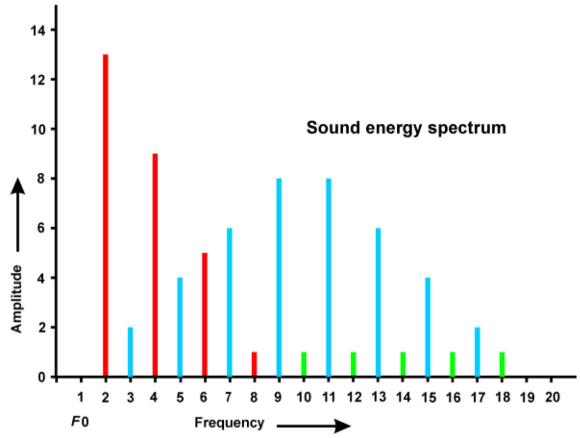

Fig. 20. Sound energy spectrum corresponding to the complex of harmonics f_0 to $9f_0$ and $1/f$ amplitude ratio. Odd harmonics [blue] are – sine functions, even harmonics [red] are cosine functions even harmonics [green] are – cosine functions.

In this spectrum it is clear that the fundamental $\boldsymbol{F0} = f_0$ is missed. The second in row: $\boldsymbol{2F0}$ is the strongest contribution and will therefore act as pitch representing frequency.

In this example the pitch is heard one octave higher than can be expected from the setup of the harmonic composition.

3. Conclusions.

All the calculations in chapter 2 are in full agreement with the pitch examples reported by De Cheveigné[3].

As could be expected the change in the form of the harmonics in the sound pressure stimuli – sine or cosine contributions – and changes in phase don't have that much influence on the intensity of the itch, except for the alternating [sine – cosine] or [cosine – sine] series. Paragraphs 2.e and 2.h show that in that cases the pitch is heard as $\boldsymbol{F0} = 2f_0$ related to the other examples, where in calculations the $\boldsymbol{F0}$ appears to be equal to f_0.

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